

**INTEGRATION****Answers**

1 **a** $= \frac{1}{8}(x-2)^8 + c$ **b** $= \frac{1}{2} \times \frac{1}{4}(2x+5)^4 + c$ **c** $= \frac{1}{3} \times \frac{6}{5}(1+3x)^5 + c$ **d** $= 4 \times \frac{1}{6}(\frac{1}{4}x-2)^6 + c$
 $= \frac{1}{8}(2x+5)^4 + c$ $= \frac{2}{5}(1+3x)^5 + c$ $= \frac{2}{3}(\frac{1}{4}x-2)^6 + c$

e $= -\frac{1}{5} \times \frac{1}{5}(8-5x)^5 + c$ **f** $= \int (x+7)^{-2} dx$ **g** $= \int 8(4x-3)^{-5} dx$ **h** $= \int \frac{1}{2}(5-3x)^{-3} dx$
 $= -\frac{1}{25}(8-5x)^5 + c$ $= -(x+7)^{-1} + c$ $= \frac{1}{4} \times \frac{8}{-4}(4x-3)^{-4} + c$ $= -\frac{1}{3} \times \frac{1}{-4}(5-3x)^{-2} + c$
 $= \frac{-1}{2(4x-3)^4} + c$ $= \frac{1}{12(5-3x)^2} + c$

2 **a** $= \frac{2}{5}(3+t)^{\frac{5}{2}} + c$ **b** $= \int (4x-1)^{\frac{1}{2}} dx$ **c** $= \frac{1}{2} \ln |2y+1| + c$
 $= \frac{1}{4} \times \frac{2}{3}(4x-1)^{\frac{3}{2}} + c$
 $= \frac{1}{6}(4x-1)^{\frac{3}{2}} + c$

d $= \frac{1}{2}e^{2x-3} + c$ **e** $= 3 \times \frac{1}{-7} \ln |2-7r| + c$ **f** $= \int (5t-2)^{\frac{1}{3}} dt$
 $= -\frac{3}{7} \ln |2-7r| + c$ $= \frac{1}{5} \times \frac{3}{4}(5t-2)^{\frac{4}{3}} + c$
 $= \frac{3}{20}(5t-2)^{\frac{4}{3}} + c$

g $= \int (6-y)^{-\frac{1}{2}} dy$ **h** $= -\frac{5}{3}e^{7-3t} + c$ **i** $= 4 \times \frac{1}{3} \ln |3u+1| + c$
 $= -2(6-y)^{\frac{1}{2}} + c$ $= \frac{4}{3} \ln |3u+1| + c$

3 **a** $f(x) = \int 8(2x-3)^3 dx$
 $= \frac{1}{2} \times 2(2x-3)^4 + c$
 $= (2x-3)^4 + c$
 $(2, 6) \Rightarrow 6 = 1 + c$
 $\therefore c = 5$
 $f(x) = (2x-3)^4 + 5$

c $f(x) = \int 2 - \frac{8}{4x-1} dx$
 $= 2x - 8 \times \frac{1}{4} \ln |4x-1| + c$
 $= 2x - 2 \ln |4x-1| + c$
 $(\frac{1}{2}, 4) \Rightarrow 4 = 1 + c$
 $\therefore c = 3$
 $f(x) = 2x - 2 \ln |4x-1| + 3$

b $f(x) = \int 6e^{2x+4} dx$
 $= 3e^{2x+4} + c$
 $(-2, 1) \Rightarrow 1 = 3 + c$
 $\therefore c = -2$
 $f(x) = 3e^{2x+4} - 2$

d $f(x) = \int 8x - 3(3x-2)^{-2} dx$
 $= 4x^2 + \frac{1}{3} \times 3(3x-2)^{-1} + c$
 $= 4x^2 + (3x-2)^{-1} + c$
 $(-1, 3) \Rightarrow 3 = 4 - \frac{1}{5} + c$
 $\therefore c = -\frac{4}{5}$
 $f(x) = 4x^2 + \frac{1}{3x-2} - \frac{4}{5}$

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4	a	$\begin{aligned} &= \left[\frac{1}{3} \times \frac{1}{3} (3x+1)^3 \right]_0^1 \\ &= \frac{1}{9} [(3x+1)^3]_0^1 \\ &= \frac{1}{9} (64 - 1) \\ &= 7 \end{aligned}$	b	$\begin{aligned} &= \left[\frac{1}{2} \times \frac{1}{4} (2x-1)^4 \right]_1^2 \\ &= \frac{1}{8} [(2x-1)^4]_1^2 \\ &= \frac{1}{8} (81 - 1) \\ &= 10 \end{aligned}$	c	$\begin{aligned} &= \int_2^4 (5-x)^{-2} dx \\ &= [(5-x)^{-1}]_2^4 \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$
	d	$\begin{aligned} &= \left[\frac{1}{2} e^{2x+2} \right]_{-1}^1 \\ &= \frac{1}{2} (e^4 - 1) \end{aligned}$	e	$\begin{aligned} &= \int_2^6 (3x-2)^{\frac{1}{2}} dx \\ &= \left[\frac{1}{3} \times \frac{2}{3} (3x-2)^{\frac{3}{2}} \right]_2^6 \\ &= \frac{2}{9} [(3x-2)^{\frac{3}{2}}]_2^6 \\ &= \frac{2}{9} (64 - 8) \\ &= 12\frac{4}{9} \end{aligned}$	f	$\begin{aligned} &= [4 \times \frac{1}{6} \ln 6x-3]_1^2 \\ &= \frac{2}{3} [\ln 6x-3]_1^2 \\ &= \frac{2}{3} (\ln 9 - \ln 3) \\ &= \frac{2}{3} \ln 3 \end{aligned}$
	g	$\begin{aligned} &= \int_0^1 (7x+1)^{-\frac{1}{3}} dx \\ &= \left[\frac{1}{7} \times \frac{3}{2} (7x+1)^{\frac{2}{3}} \right]_0^1 \\ &= \frac{3}{14} (4 - 1) \\ &= \frac{9}{14} \end{aligned}$	h	$\begin{aligned} &= \left[\frac{1}{5} \ln 5x+3 \right]_{-7}^{-1} \\ &= \frac{1}{5} (\ln 2 - \ln 32) \\ &= \frac{1}{5} (\ln 2 - 5 \ln 2) \\ &= -\frac{4}{5} \ln 2 \end{aligned}$	i	$\begin{aligned} &= \frac{1}{8} \int_4^7 (x-4)^3 dx \\ &= \frac{1}{8} \left[\frac{1}{4} (x-4)^4 \right]_4^7 \\ &= \frac{1}{32} (81 - 0) \\ &= 2\frac{17}{32} \end{aligned}$
5	a	$\begin{aligned} &= \int_3^4 e^{3-x} dx \\ &= [-e^{3-x}]_3^4 \\ &= -e^{-1} - (-1) \\ &= 1 - \frac{1}{e} \end{aligned}$	b	$\begin{aligned} &= \int_2^3 (3x-5)^3 dx \\ &= \left[\frac{1}{3} \times \frac{1}{4} (3x-5)^4 \right]_2^3 \\ &= \frac{1}{12} (256 - 1) \\ &= 21\frac{1}{4} \end{aligned}$	d	$\begin{aligned} &= \int_{-2}^0 (1-2x)^{-2} dx \\ &= \left[-\frac{1}{2} \times -(1-2x)^{-1} \right]_{-2}^0 \\ &= \frac{1}{2} (1 - \frac{1}{5}) \\ &= \frac{2}{5} \end{aligned}$
	c	$\begin{aligned} &= \int_1^4 \frac{3}{4x+2} dx \\ &= [3 \times \frac{1}{4} \ln 4x+2]_1^4 \\ &= \frac{3}{4} (\ln 18 - \ln 6) \\ &= \frac{3}{4} \ln 3 \end{aligned}$	6	$\begin{aligned} &= \int_0^1 12(2x+1)^{-3} dx \\ &= \left[\frac{1}{2} \times (-6)(2x+1)^{-2} \right]_0^1 \\ &= \left[\frac{-3}{(2x+1)^2} \right]_0^1 \\ &= -\frac{1}{3} - (-3) \\ &= \frac{8}{3} \end{aligned}$		